Exercise 4G

1 $y=\frac{1}{3}x^{\frac{3}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}x^{\frac{1}{2}}$

Using $s=\int_{x_A}^{x_B}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ gives:

$$s=\int_0^{12}\sqrt{1+\left(\frac{1}{2}x^{\frac{1}{2}}\right)^2}\,dx$$

$$s=4\int_1^4\sqrt{u}\,du=\int_0^{12}\sqrt{1+\frac{1}{4}x}\,dx$$

Let $u=1+\frac{1}{4}x \Rightarrow \frac{du}{dx}=\frac{1}{4} \Rightarrow dx=4du$

When $x=0$, $u=1$

When $x=12$, $u=4$

Therefore:

$$=4\times\frac{2}{3}\left[u^{\frac{3}{2}}\right]_1^4$$

$$=\frac{8}{3}(8-1)$$

$$=\frac{56}{3}$$

2 $y=\ln\cos x \Rightarrow \frac{dy}{dx}=-\tan x$

Using $s=\int_{x_A}^{x_B}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ gives:

$$s=\int_0^{\frac{\pi}{3}}\sqrt{1+(-\tan x)^2}\,dx$$

$$=\int_0^{\frac{\pi}{3}}\sqrt{1+\tan^2 x}\,dx$$

$$=\int_0^{\frac{\pi}{3}}\sec x\,dx$$

$$=\left[\ln(\sec x+\tan x)\right]_0^{\frac{\pi}{3}}$$

$$=\ln\left(2+\sqrt{3}\right)-0$$

$$=\ln\left(2+\sqrt{3}\right)$$

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3 $y = 2\cosh\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \sinh\left(\frac{x}{2}\right)$

Using $s = \int_{x_A}^{x_B} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$$s = \int_0^{\ln 4} \sqrt{1+\left(\sinh\left(\frac{x}{2}\right)\right)^2}\, dx$$

$$= \int_0^{\ln 4} \sqrt{1+\sinh^2\left(\frac{x}{2}\right)}\, dx$$

$$= \int_0^{\ln 4} \sqrt{\cosh^2\left(\frac{x}{2}\right)}\, dx$$

$$= \int_0^{\ln 4} \cosh\left(\frac{x}{2}\right) dx$$

Let $u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2} \Rightarrow dx = 2du$

When $x = 0$, $u = 0$

When $x = \ln 4$, $u = \ln 2$

Therefore:

$$s = 2\int_0^{\ln 2} \cosh u\, du$$

$$= 2\left[\sinh u\right]_0^{\ln 2}$$

$$= \frac{2}{2}\left(e^{\ln 2} - e^{-\ln 2}\right)$$

$$= 2 - \frac{1}{2}$$

$$= \frac{3}{2}$$

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4 $y^2 = \frac{4}{9}x^3$

$y = \frac{2}{3}x^{\frac{3}{2}} \Rightarrow \frac{dy}{dx} = \pm x^{\frac{1}{2}}$

Using $s = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$s = \int_0^3 \sqrt{1 + \left(\pm x^{\frac{1}{2}}\right)^2}\, dx$

$= \int_0^3 \sqrt{1 + x}\, dx$

Let $u = 1 + x \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

When $x = 0$, $u = 1$

When $x = 3$, $u = 4$

Therefore:

$s = \int_1^4 \sqrt{u}\, du$

$= \frac{2}{3}\left[u^{\frac{3}{2}}\right]_1^4$

$= \frac{2}{3}(8 - 1)$

$= \frac{14}{3}$

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5 $y = \frac{1}{2}\sinh^2 2x$

Let $u = 2x \Rightarrow \frac{du}{dx} = 2$

$y = \frac{1}{2}\sinh^2 u \Rightarrow \frac{dy}{du} = \sinh u \cosh u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

Therefore:

$\frac{dy}{dx} = 2\sinh u \cosh u$

$= 2\sinh(2x)\cosh(2x)$

Since $\sinh 2x = 2\sinh x \cosh x$

$\frac{dy}{dx} = \sinh(4x)$

Using $s = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$s = \int_0^1 \sqrt{1 + \sinh^2 4x}\, dx$

$= \int_0^1 \cosh 4x\, dx$

$= \frac{1}{4}[\sinh 4x]_0^1$

$= \frac{1}{4}\left[\frac{e^{4x} - e^{-4x}}{2}\right]_0^1$

$= \frac{1}{8}\left[(e^4 - e^{-4}) - (e^0 - e^0)\right]$

$= \frac{1}{8}\left(e^4 + \frac{1}{e^4}\right)$

$= \frac{e^8 + 1}{8e^4}$

$= 6.83$ (3 s.f.)

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6 $y=\frac{1}{4}\left(2x^2-\ln x\right)\Rightarrow\frac{dy}{dx}=\frac{1}{4}\left(4x-\frac{1}{x}\right)$

Using $s=\int_{x_A}^{x_B}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ gives:

$$s=\int_1^2\left(\sqrt{1+\left(\frac{1}{4}\left(4x-\frac{1}{x}\right)\right)^2}\right)dx$$

$$=\int_1^2\left(\sqrt{1+\left(x-\frac{1}{4x}\right)^2}\right)dx$$

$$=\int_1^2\left(\sqrt{1+\left(x^2-\frac{1}{2}+\frac{1}{16x^2}\right)}\right)dx$$

$$=\int_1^2\sqrt{x^2+\frac{1}{16x^2}+\frac{1}{2}}\,dx$$

$$=\int_1^2\sqrt{\left(x+\frac{1}{4x}\right)^2}\,dx$$

$$=\int_1^2\left(x+\frac{1}{4x}\right)dx$$

$$=\int_1^2 x\,dx+\frac{1}{4}\int_1^2\frac{1}{x}\,dx$$

$$=\frac{1}{2}\left[x^2\right]_1^2+\frac{1}{4}\left[\ln x\right]_1^2$$

$$=\frac{1}{2}(4-1)+\frac{1}{4}(\ln 2-\ln 1)$$

$$=\frac{3}{2}+\frac{1}{4}\ln 2$$

$$=\frac{1}{4}(6+\ln 2)$$

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7 $y = 2\text{arcosh}\left(\frac{x}{2}\right)$

$$\frac{dy}{dx} = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{\left(\frac{x}{2}\right)^2 - 1}} = \frac{2}{\sqrt{x^2 - 4}}$$

Using $s = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$$s = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{2}{\sqrt{x^2 - 4}}\right)^2}\, dx$$

$$= \int_{x_A}^{x_B} \sqrt{1 + \frac{4}{x^2 - 4}}\, dx$$

$$= \int_{x_A}^{x_B} \sqrt{\frac{(x^2 - 4) + 4}{x^2 - 4}}\, dx$$

$$= \int_{x_A}^{x_B} \frac{x}{\sqrt{x^2 - 4}}\, dx$$

$$= \frac{1}{2} \int_{x_A}^{x_B} 2x(x^2 - 4)^{-\frac{1}{2}}\, dx$$

$$= \frac{1}{2}\left[\frac{1}{\left(\frac{1}{2}\right)}(x^2 - 4)^{\frac{1}{2}}\right]_{x_A}^{x_B} = \left[(x^2 - 4)^{\frac{1}{2}}\right]_{x_A}^{x_B}$$

When the curve crosses the x-axis, $y = 0$, and

$$0 = 2\text{arcosh}\left(\frac{x}{2}\right)$$

$$\frac{x}{2} = \cosh 0$$

$$x = 2$$

$$s = \left[(x^2 - 4)^{\frac{1}{2}}\right]_2^{\frac{5}{2}}$$

$$= \left(\left(\frac{5}{2}\right)^2 - 4\right)^{\frac{1}{2}} - (2^2 - 4)^{\frac{1}{2}}$$

$$= \sqrt{\frac{5^2 - 4 \times 4}{4}} - \sqrt{0}$$

$$= \sqrt{\frac{9}{4}}$$

$$= \frac{3}{2}$$

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In Example 25:

$$x = t + \frac{1}{t} \text{ and } y = 2\ln t$$

Therefore:

$$t = e^{\frac{y}{2}} \Rightarrow x = e^{\frac{y}{2}} + \frac{1}{e^{\frac{y}{2}}}$$

$$x = 2\cosh\left(\frac{y}{2}\right)$$

Therefore:

$$\frac{y}{2} = \text{arcosh}\left(\frac{x}{2}\right) \Rightarrow y = 2\text{arcosh}\left(\frac{x}{2}\right)$$

The same as in question 7.

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8 $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

Using $s = \int_{x_A}^{x_B} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ and the symmetry of the curve gives:

$$s = \int_{-2}^{2} \left(\sqrt{1+(2x)^2}\right)dx = 2\int_{0}^{2} \left(\sqrt{1+(2x)^2}\right)dx$$

$$= 2\int_{0}^{2} \sqrt{1+4x^2}\,dx$$

Let $x = \frac{1}{2}\sinh\theta \Rightarrow \frac{dx}{d\theta} = \frac{1}{2}\cosh\theta$

When $x = 0,\ \theta = 0$

When $x = 2,\ \theta = \text{arsinh}(4)$

$$= \frac{1}{4}(6+\ln 2)$$

$$s = 2\int_{0}^{\text{arsinh}(4)} \sqrt{1+\sinh^2\theta} \times \frac{1}{2}\cosh\theta\,d\theta$$

$$= \int_{0}^{\text{arsinh}(4)} \sqrt{\cosh^2\theta} \times \cosh\theta\,d\theta$$

$$= \int_{0}^{\text{arsinh}(4)} \cosh^2\theta\,d\theta$$

$$= \frac{1}{2}\int_{0}^{\text{arsinh}(4)} (\cosh 2\theta + 1)\,d\theta$$

$$= \frac{1}{2}\left[\frac{1}{2}\sinh 2\theta + \theta\right]_0^{\text{arsinh}(4)}$$

$$= \frac{1}{4}[\sinh 2\theta]_0^{\text{arsinh}(4)} + \frac{1}{2}\ln\left(4+\sqrt{4^2+1}\right) - 0$$

$$= \frac{1}{4}[2\sinh\theta\cosh\theta]_0^{\text{arsinh}(4)} + \frac{1}{2}\ln\left(4+\sqrt{17}\right)$$

$$= \frac{1}{4}\left[2\sinh\theta\sqrt{\sinh^2\theta+1}\right]_0^{\text{arsinh}(4)} + \frac{1}{2}\ln\left(4+\sqrt{17}\right)$$

$$= \frac{1}{4}\left(2\times 4\sqrt{4^2+1} - 0\right) + \frac{1}{2}\ln\left(4+\sqrt{17}\right)$$

$$= 2\sqrt{4^2+1} + \frac{1}{2}\ln\left(4+\sqrt{17}\right)$$

$$= 2\sqrt{17} + \frac{1}{2}\ln\left(4+\sqrt{17}\right)$$

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9 $x = r\cos\theta \Rightarrow \frac{dx}{d\theta} = -r\sin\theta$

$y = r\sin\theta \Rightarrow \frac{dy}{d\theta} = r\cos\theta$

Using $s = \int_{\theta_A}^{\theta_B} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$ gives:

$$s = \int_0^{2\pi} \sqrt{r^2\sin^2\theta + r^2\cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{r^2\left(\sin^2\theta + \cos^2\theta\right)}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{r^2}\, d\theta$$

$$= \int_0^{2\pi} r\, d\theta$$

$$= \left[r\theta\right]_0^{2\pi}$$

$= 2\pi r$ as required

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10 $x = 2a\cos^3 t \Rightarrow \frac{dx}{dt} = -6a\cos^2 t\sin t$

$y = 2a\sin^3 t \Rightarrow \frac{dy}{dt} = 6a\sin^2 t\cos t$

Using $s = \int_{t_A}^{t_B} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ gives:

$$s = \int_0^{\frac{\pi}{2}} \sqrt{\left(-6a\cos^2 t\sin t\right)^2 + \left(6a\sin^2 t\cos t\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{36a^2\cos^4 t\sin^2 t + 36a^2\sin^4 t\cos^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{36a^2\left(\cos^4 t\sin^2 t + \sin^4 t\cos^2 t\right)}\, dt$$

$$= 6a\int_0^{\frac{\pi}{2}} \sqrt{\cos^2 t\sin^2 t\left(\cos^2 t + \sin^2 t\right)}\, dt$$

$$= 6a\int_0^{\frac{\pi}{2}} \sqrt{\cos^2 t\sin^2 t}\, dt$$

$$= 6a\int_0^{\frac{\pi}{2}} \cos t\sin t\, dt$$

$$= 6a\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 2t\, dt$$

$$= 3a\int_0^{\frac{\pi}{2}} \sin 2t\, dt$$

$$= \frac{3a}{2}\left[-\cos 2t\right]_0^{\frac{\pi}{2}}$$

$$= \frac{3a}{2}\left[\left(-\cos\pi\right) - \left(-\cos 0\right)\right]$$

$$= \frac{3a}{2}\left[1+1\right]$$

$= 3a$ as required

The arc AB has length $3a$.
There are four of these arcs, therefore the total length is $12a$.

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11 $x = \tanh u \Rightarrow \frac{dx}{du} = \text{sech}^2 u$

$y = \text{sech}\, u \Rightarrow \frac{dy}{du} = -\text{sech}\, u \tanh u$

Using $s = \int_{u_A}^{u_B} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$ gives:

$$s = \int_0^1 \sqrt{\left(\text{sech}^2 u\right)^2 + \left(-\text{sech}\, u \tanh u\right)^2}\, du$$

$$= \int_0^1 \sqrt{\text{sech}^4 u + \text{sech}^2 u \tanh^2 u}\, du$$

$$= \int_0^1 \sqrt{\text{sech}^2 u \left(\text{sech}^2 u + \tanh^2 u\right)}\, du$$

$$= \int_0^1 \sqrt{\text{sech}^2 u}\, du$$

$$= \int_0^1 \text{sech}\, u\, du$$

$$= \int_0^1 \frac{1}{\cosh u}\, du$$

$$= 2\int_0^1 \frac{1}{e^u + e^{-u}}\, du$$

$$= 2\int_0^1 \frac{e^u}{e^{2u} + 1}\, du$$

Let $p = e^u \Rightarrow dp = e^u du$

When $u = 0$, $p = 1$

When $u = 1$, $p = e$

$$s = 2\int_1^e \frac{1}{p^2 + 1}\, dp$$

$$= 2\left[\arctan p\right]_1^e$$

$$= 2\left(\arctan e - \arctan 1\right)$$

$$= 2\left(\arctan e - \frac{\pi}{4}\right)$$

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12 $x = a(\theta + \sin\theta) \Rightarrow \dfrac{dx}{d\theta} = a(1+\cos\theta)$

$y = a(1-\cos\theta) \Rightarrow \dfrac{dy}{d\theta} = a\sin\theta$

Using $s = \int_{\theta_A}^{\theta_B} \sqrt{\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2}\, d\theta$ gives:

$$s = \int_0^{\pi} \sqrt{\left(a(1+\cos\theta)\right)^2 + (a\sin\theta)^2}\, du$$

$$= \int_0^{\pi} \sqrt{a^2(1+\cos\theta)^2 + a^2\sin^2\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{a^2\left(1+2\cos\theta+\cos^2\theta\right) + a^2\sin^2\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{a^2 + 2a^2\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{a^2 + 2a^2\cos\theta + a^2\left(\cos^2\theta + \sin^2\theta\right)}\, d\theta$$

$$= \int_0^{\pi} \sqrt{2a^2 + 2a^2\cos\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{2a^2(1+\cos\theta)}\, d\theta$$

$$= \int_0^{\pi} \sqrt{4a^2\left(\frac{1+\cos\theta}{2}\right)}\, d\theta$$

$$= 2a\int_0^{\pi} \sqrt{\cos^2\left(\frac{\theta}{2}\right)}\, d\theta$$

$$= 2a\int_0^{\pi} \cos\left(\frac{\theta}{2}\right) d\theta$$

$$= 2a\left[2\sin\left(\frac{\theta}{2}\right)\right]_0^{\pi}$$

$$= 4a\left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right]$$

$$= 4a$$

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13 $x = t + \sin t \Rightarrow \frac{dx}{dt} = 1 + \cos t$

$y = 1 - \cos t \Rightarrow \frac{dy}{dt} = \sin t$

Using $s = \int_{t_A}^{t_B} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ gives:

$$s = \int_0^{\frac{\pi}{3}} \sqrt{(1+\cos t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{1 + 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{2 + 2\cos t}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{2(1+\cos t)}\, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{4\left(\frac{1+\cos t}{2}\right)}\, dt$$

$$= 2\int_0^{\frac{\pi}{3}} \sqrt{\cos^2\left(\frac{t}{2}\right)}\, dt$$

$$= 2\int_0^{\frac{\pi}{3}} \cos\left(\frac{t}{2}\right) dt$$

$$= 2\left[2\sin\left(\frac{t}{2}\right)\right]_0^{\frac{\pi}{3}}$$

$$= 4\left[\sin\left(\frac{\pi}{6}\right) - \sin(0)\right]$$

$$= 2$$

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14 $x = e^t \cos t \Rightarrow \frac{dx}{dt} = -e^t \sin t + e^t \cos t$

$y = e^t \sin t \Rightarrow \frac{dy}{dt} = e^t \cos t + e^t \sin t$

Using $s = \int_{t_A}^{t_B} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ gives:

$$s = \int_0^{\frac{\pi}{4}} \sqrt{\left(-e^t \sin t + e^t \cos t\right)^2 + \left(e^t \cos t + e^t \sin t\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\left(e^{2t} \sin^2 t - 2e^{2t} \sin t \cos t + e^{2t} \cos^2 t\right) + \left(e^{2t} \cos^2 t + 2e^{2t} \sin t \cos t + e^{2t} \sin^2 t\right)}\, dt$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{2e^{2t} \sin^2 t + 2e^{2t} \cos^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{2e^{2t}\left(\sin^2 t + \cos^2 t\right)}\, dt$$

$$= \sqrt{2}\int_0^{\frac{\pi}{4}} \sqrt{e^{2t}}\, dt$$

$$= \sqrt{2}\int_0^{\frac{\pi}{4}} e^t\, dt$$

$$= \sqrt{2}\left[e^t\right]_0^{\frac{\pi}{4}}$$

$$= \sqrt{2}\left(e^{\frac{\pi}{4}} - e^0\right)$$

$$= \sqrt{2}\left(e^{\frac{\pi}{4}} - 1\right)$$

15 a $y = \sqrt{3} \sin \Rightarrow \frac{dy}{dx} = \sqrt{3} \cos x$

Using $s = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$$s = \int_0^{\frac{\pi}{2}} \sqrt{1 + \left(\sqrt{3} \cos x\right)^2}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1 + 3\cos^2 x}\, dx$$

Since there are 3 more regions with this length:

$s = 4\int_0^{\frac{\pi}{2}} \sqrt{1 + 3\cos^2 x}\, dx$ as required

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15 b $x = \cos t \Rightarrow \dfrac{dx}{dt} = -\sin t$

$y = 2\sin t \Rightarrow \dfrac{dy}{dt} = 2\cos t$

Using $s = \displaystyle\int_{t_A}^{t_B} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ gives:

$$s = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (2\cos t)^2}\, dt$$
$$= \int_0^{2\pi} \sqrt{\sin^2 t + 4\cos^2 t}\, dt$$
$$= \int_0^{2\pi} \sqrt{1 - \cos^2 t + 4\cos^2 t}\, dt$$
$$= \int_0^{2\pi} \sqrt{1 + 3\cos^2 t}\, dt$$
$$= 4\int_0^{\frac{\pi}{2}} \sqrt{1 + 3\cos^2 t}\, dt \text{ as required}$$

Challenge

The integrand $\sqrt{t^3 - 1}$ is the gradient of f(x) expressed in terms of t.
Thus:

$$\sqrt{t^3 - 1} \Rightarrow \sqrt{x^3 - 1} = f'(x) = \frac{dy}{dx}$$

Hence using $s = \displaystyle\int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ gives:

$$s = \int_1^4 \sqrt{1 + \left(\sqrt{x^3 - 1}\right)^2}\, dx$$
$$= \int_1^4 \sqrt{1 + x^3 - 1}\, dx$$
$$= \int_1^4 \sqrt{x^3}\, dx$$
$$= \int_1^4 x^{\frac{3}{2}}\, dx$$
$$= \frac{2}{5}\left[x^{\frac{5}{2}}\right]_1^4$$
$$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right)$$
$$= \frac{2}{5}(32 - 1)$$
$$= \frac{62}{5}$$

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